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Curriculum I.12D
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ONTARIO DEPARTMENT OF EDUCATION

MATHEMATICS

INTERMEDIATE DIVISION

Grade 10

Five-Year Program

For use in Grade 10 for students who have completed Curriculum I.12A, I.12B and I.12C

ISSUED BY AUTHORITY OF
THE MINISTER OF EDUCATION

MATHEMATICS

Introduction

1. This course is designed for pupils in the Five-year Programs who have completed successfully the earlier courses in the series — Curriculum I.12A, I.12B, and I.12C (Five-year Program).

2. The course consists of two parts: a core, which is to be studied by all students; and optional topics from which a selection will be made to complete the year's work. The compulsory part of the course consists of:
   UNIT 1: The System of Real Numbers
   UNIT 2: Relations and Functions
   UNIT 3: Geometry

   These units require approximately twenty-five weeks of teaching time.

   Unit 1, The System of Real Numbers, provides a review of the number systems previously studied. This review is introduced by developing a modular or other finite system. The properties of groups and fields are established; group and field structure within the various number systems is investigated. This unit also provides further study of the irrational numbers.

   Unit 2, Relations and Functions, introduces these concepts with emphasis on first degree functions. Systems of first degree equations and the analytic geometry related to $y = mx + b$ are also included.

   Unit 3, Geometry, provides a review of and extends the study of the properties of geometric figures established in earlier grades and introduces formal deductive reasoning and the nature of proof.

   Four optional topics are provided of which *at least one* is to be studied. These topics are:
   UNIT 4: Topics from the History of Mathematics
   UNIT 5: The Nature of Proof
   UNIT 6: Mechanical Aids to Computation
   UNIT 7: Algorithms

These units each require approximately two weeks of teaching time.

Topics from the History of Mathematics, Unit 4, should not be taught in a block, but rather the topics should be introduced at appropriate times throughout the course.

Although it is understood that frequent reference to history should be made in the introduction of many mathematics topics, Unit 4 provides the necessary time to allow these introductions to be extended.

3. "Suggestions for Teachers" are included with this course outline. These notes provide an interpretation of the intent of the course and suggestions for classroom presentation but should not inhibit originality of approach by the teacher nor adaptation of this course to local needs.

UNIT 1:
THE SYSTEM OF REAL NUMBERS (5 weeks)

1. A Review of the Systems of Natural Numbers, Whole Numbers, Integers, and Rational Numbers (2 weeks)

   This review should be introduced by the development of a modular or other finite system. Such a development should provide:

   (i) the experience of deciding on the definition of an operation; the establishment of the operation table

   (ii) review of the ideas of closure, commutativity, and associativity

   (iii) review of identity and inverse elements

   (iv) an opportunity for the introduction of the group as a basic mathematical structure. (Also the Abelian or commutative group)

   The comparison of this group structure with the structure of the natural numbers, the whole numbers, the integers, and the rational numbers under addition or multiplication as defined on these sets; a review of the definitions of these operations

   The definition of a second operation on the finite set

providing further review of group structure and leading to the properties of a field

The comparison of the field properties with those of the above number systems completes the review of these systems

In the exercises throughout this section, other examples of these structures should be introduced, such as: a permutation group; the additive group of 2×2 matrices

2. THE REAL NUMBER SYSTEM (1 week)
Representation of rational numbers in decimal notation

The existence of irrational numbers

(i) decimal notation and operations

(ii) demonstration that radicals such as $\sqrt{2}$ are not rational numbers

(iii) representation on a number line

The set of real numbers as the union of the sets of rational and irrational numbers

Operations, as defined on the set of rational numbers, extended to the set of real numbers.

The real number field; comparison with the field of arithmetic modulo p where p is a prime number

Order in the set of real numbers; real number line

3. OPERATIONS WITH RADICALS (2 weeks)
Review of previous work with radicals

Multiplication of binomial expressions involving radicals

Manipulation of radical expressions using conjugate radicals

Problems involving use of square root, square root tables

The field with elements of the form $a + b\sqrt{2}$, where a and b are rational numbers

UNIT 2:
RELATIONS AND FUNCTIONS (8 weeks)

1. INTRODUCTION TO BINARY RELATIONS AND FUNCTIONS (2 weeks)

Non-numerical as well as numerical examples of relations: pairing of elements of two sets; arrow diagrams, ordered pairs, domain, range

Functions as special relations

Algebraic defining sentences

Cartesian products, graphing in the Cartesian plane Relations and functions as subsets of Cartesian products

Relations and functions as subsets of Cartesian products

Vertical line test for a function

2. First Degree Relations (1½ weeks)
Functions defined by $y = mx + b$; graph, x-, y- intercepts

Graphs of relations defined by $y < mx + b$, $y > mx + b$, $y \leqq mx + b$, $y \geqq mx + b$

Graphs of relations involving union and intersection

3. Systems of First Degree Equations (3 weeks)
Solution of systems of equations in two variables
(a) graphically
(b) algebraically: (i) by comparison
(ii) by substituion

Solution of systems by elimination by addition and subtraction showing equivalent systems

Word problems solved by first degree equations in two variables emphasizing the domain of the discussion

(Supplementary) Application of graphical solution of systems of linear inequations to simple problems in linear programming

(Supplementary) Use of 2×2 matrices in solving two equations of the first degree in two variables

4. Slope, Equations of Lines, and Applications (1½ weeks)

Slope of a line segment

Role of m and b in equations of the form $y = mx + b$

Determination of equations of lines with given slope and through a particular point using $y = mx + b$

Determination of equations of lines through two points using $y = mx + b$

Problems involving lengths, intersections, slopes and equations of sides of rectilinear figures

UNIT 3:
GEOMETRY (12 weeks)

1. DEDUCTION IN GEOMETRY (4½ weeks)

In this topic the nature of a deductive proof is established. Congruency and parallelism facts established in earlier grades are to be used as the basis for deductive proofs of further properties of triangles and polygons. It is not intended that there be a formal organization of Euclidean Geometry, but rather that short sequences of related theorems be developed.

Deductions based on congruency and parallelism to illustrate the nature of deductive proof

The sum of angles of polygons deduced from the angle sum of a triangle

Regular polygons, symmetry

Parallelogram properties

Proof of Pythagorean theorem based on dissection of a square

2. THREE-DIMENSIONAL GEOMETRY (1½ weeks)

Emphasis should be placed on the use of models and on the diagrammatic representation of solids throughout this topic.

Points, lines and planes

Parallel lines, skew lines, line perpendicular to a plane, line parallel to a plane, perpendicular and parallel planes and their diagrammatic representation

Definition of the regular polyhedra, face and body diagonals, Euler's formula

3. TRANSFORMATIONS (3 weeks)

Effect of translations, rotations and reflections on simple geometric figures

Composition of transformations in simple cases

Dilatation, similarity, simple deductions on similarity

Examples of the group of symmetries: equilateral triangle, rectangle, square, rhombus

Translations and reflections of points, lines and simple figures in a Cartesian coordinate system

4. VECTORS (3 weeks)

Vectors as directed line segments

Equivalent vectors

Addition of vectors, parallelogram and triangle of addition

Multiplication of a vector by a scalar

Geometric deductions by vector methods

Applications to physics, e.g. navigation problems

UNIT 4:
HISTORY OF MATHEMATICS (2 weeks)

The inclusion of this unit is based on the premise that mathematics is taught with greater clarity and relevance and is better able to capture the interest and imagination if students have an opportunity to become acquainted with the individuals who were involved in the development of some of the great mathematical ideas. It is not intended that this should be a formal discourse on the history of mathematics.

It is intended that emphasis should be placed on the discussion of personalities and appropriate anecdotes.

This unit should not be taught as a topic by itself but the different items should be inserted into the course at suitable times throughout the year. This approach lends itself to student participation either by way of a student organized and directed seminar dealing with a particular mathematical idea and the persons involved, or a biograph-

ical sketch of a leading mathematician and his contribution to mathematics.

The following is a list of suggested topics and their relationship to the units of the course.

1. Real Numbers
   Irrational numbers: Pythagoras
   Decimal Notation
   History of the radical symbol
   Number line, real numbers: Dedekind
   Set theory: Cantor, Hilbert, Brouwer
   History and evaluation of π
   Groups and Fields: Abel, Galois, Klein, Cauchy
2. Relations and Functions
   Newton, Leibniz, Descartes, Cayley
3. Geometry
   Ancient: Pythagoras, Euclid, Plato, Aristotle, Zeno, Archimedes
   Modern: Gauss, Riemann, Lobachevsky, Euler
4. The Nature of Proof
   Euclid, Whitehead and Russell, George Boole, Lewis Carroll
5. Mechanical Aids to Computation
   History of the abacus
   Napier, Pascal, Leibniz, Von Neumann
   Computing machines

UNIT 5:
THE NATURE OF PROOF (2 weeks)

This unit provides a formal introduction to the nature of proof. It is not intended that this unit be used as an introduction to the geometry part of this course but rather that the geometry topics will provide an introduction to deduction and thus supply illustrative examples for this unit.

Statements and sentential connectives:

Definition of statements, connectives, implications, compound statements

Converse, biconditional, equivalent statements

Deductive reasoning, law of detachment

Simple deductive arguments

Analysis

The quantifier "all"; syllogism

Contradictory statements; indirect proof

UNIT 6:
MECHANICAL AIDS TO COMPUTATION (2 weeks)

This unit is not intended to produce "experts" in the use of the slide rule or mechanical calculators, but rather to acquaint the students with the basic fundamentals so that they may be able to use these devices as tools in their study of mathematics and science.

Since many of the ideas involved are common to both the slide rule and the calculator, it would seem advisable to consider them simultaneously. However, as the number of calculators available for use may be limited, practice on the machines may have to be on a rotation basis. (The use of programmed materials might prove helpful in the teaching of the slide rule.)

Use of standard notation for placing decimal points

Estimation of results

Approximations: rounding

Use of a mechanical calculator for addition and subtraction

Use of a slide rule and a mechanical calculator for multiplication, division, squares, and square roots

Repeated multiplication and repeated division

Evaluation of expressions involving both products and quotients

Application to problems involving products, quotients, proportions, powers, roots

UNIT 7:
ALGORITHMS (2 weeks)

This topic is designed to familiarize the student with algorithms. Many of the basic procedures in mathematics

already familiar to the student are examples of algorithms. These previously encountered procedures were studied for their applications and not as illustrations of a "procedure method". At this time the "procedure method" itself will be studied formally. This topic also is linked very closely to computer-oriented mathematics and an understanding of an algorithm is essential to programming for a computer.

Description and definition of an algorithm as a precise sequence of instructions used to solve a problem

Examples of simple algorithms for topics previously studied written as a series of instructions (in English or in mathematical form), e.g., long division, square root (any method), summing a series, summation of data to find the mean, tabulating values of an expression (function)

Expression of an algorithm in flow chart form: looping, branching, checking

SUGGESTIONS FOR TEACHERS

The following set of notes is included to provide an indication of the intent of the course of study and to provide some suggestions for presentation of the course.

These notes are in no way intended to inhibit originality of presentation, nor adaptation of the course to local situations.

UNIT 1:
THE SYSTEM OF REAL NUMBERS

1. A Review of the Systems of Natural Numbers, Whole Numbers, Integers and Rational Numbers
   The objective of this section is to emphasize the structure of a group, a number system, and a field. Examples such as:
   (a) modular arithmetic under addition
   (b) symmetries of a geometric figure, e.g., equilateral triangle
   (c) permutations of 3 elements
   (d) transformation $x \rightarrow x + h$
   (e) 2×2 matrices under addition

can be used to introduce the properties of a group. In each case it is essential to determine which of the systems form a group, that is, which systems possess the following properties:

closure
associativity
existence of an identity (neutral) element
existence of an inverse element for each element

After the properties of a group have been established the student should determine which sets of numbers previously studied (natural numbers, whole numbers, fractional numbers, integers and rational numbers) form a group under addition and which sets form a group under multiplication.

Having considered additive and multiplicative groups, the student is ready to see the relationship between these structures and a field.

The only set of numbers previously studied that forms a field is the set of rational numbers.

2. The Real Number System

The decimal notation for rational numbers, both terminating and recurring decimal forms, should be reviewed. The existence of irrational numbers can be postulated by considering:

(a) decimal forms which are not rational numbers, e.g., 0.121121112 . . ., 0.1234567891011 . . .;
(b) a decimal point followed by random digits found by using a numbered spinner, by rolling a die, by using the last digit of the telephone numbers on pages chosen at random from a telephone book;
(c) solution of $x^2 = 2$.

It is not anticipated that the student should learn a proof for "$\sqrt{2}$ is not a rational number", but that a form of proof should be given in class to convince the student that a type of number different from the rational numbers does exist. The extension of this proof should be made for other numbers, e.g., $\sqrt{3}, \sqrt{5}$. It should be also shown that there is a position on the number line for each of the irrational numbers. Arithmetic operations with irrational numbers should be attempted only by using rational approximations. The

set of real numbers is composed of the sets of rational and irrational numbers. These may be represented by the set of all decimals. Illustrations should be used to show that the field properties are satisfied by the real numbers.

The real number system illustrates an infinite field. Examples of finite fields should be given. These include numbers module p where p is a prime. Using $p = 5$ the following tables can be used to show field properties:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

The student should be able to set up a table for $p = 6$ and discover why the numbers do not form a field (lacks a unique multiplicative inverse).

3. Operations with Radicals

The student should be made aware that a terminating decimal for irrational numbers expresses only a rational approximation. It is intended that the work be confined to quadratic radicals although it should be emphasized that this is not the only type of irrational number. This section should give practice in the manipulative skills involved with quadratic radicals, e.g.,

(a) $\sqrt{2}\sqrt{3} = \sqrt{6}$; $2\sqrt{3} = \sqrt{12}$; $\sqrt{18} = 3\sqrt{2}$.

(b) $(2\sqrt{3} + \sqrt{5}) + (\sqrt{3} - 2\sqrt{5})$;
$(\sqrt{5} + \sqrt{2})\ (\sqrt{5} - \sqrt{2})$.

(c) $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \dfrac{5 + 2\sqrt{6}}{1}$ or $\dfrac{1}{5 - 2\sqrt{6}}$

(In some cases it is desirable to produce a rational numerator.)

The student has learned how to find a decimal approximation to the square root of a positive real number; tables should now be available to be used in problems involving square root. These problems can arise from solving equations resulting from formulas involving

squares, e.g., $A = \pi r^2$; $d = \sqrt{x^2 + y^2}$. He should also be able to determine the distance between two points plotted in a Cartesian plane. Undue emphasis on formulas should be avoided.

From the manipulative skills developed with quadratic radicals and the properties of a field we can now show that numbers of the form $a + b\sqrt{2}$, where a and b are rational numbers, form a field.

Addition:
$(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a + c) + (b + d)\sqrt{2}$
Closure, commutativity, associativity are evident. The identity element is $0 + 0\sqrt{2}$ and the inverse of $a + b\sqrt{2}$ is $-(a + b\sqrt{2})$.

Multiplication:
$(a + b\sqrt{2})\ (c + d\sqrt{2}) = (ac + 2bd) + (bc + ad)\sqrt{2}$
Closure, commutativity, associativity are evident. The identity element is $1 + 0\sqrt{2}$ and the inverse of $a + b\sqrt{2}$ is

$$\frac{a}{a^2 - 2b^2} + \frac{-b}{a^2 - 2b^2}\sqrt{2}$$

This can be extended to include numbers of the form $a + b\sqrt{3}$, $a + b\sqrt{5}$ and $a + b\sqrt{6}$. The student should now be aware of a new type of number field. He should also realize that there are many number systems that have field structure and many that have group structure.

UNIT 2:
RELATIONS AND FUNCTIONS

1. Introduction to Binary Relations and Functions
After a student understands what is meant by an ordered pair, domain and range, relation and function, he learns to graph special sets of ordered pairs called Cartesian products. He then *discovers* that the graphs of sets of ordered pairs of natural numbers or of integers consist of isolated points that lie in a straight line or in a region.

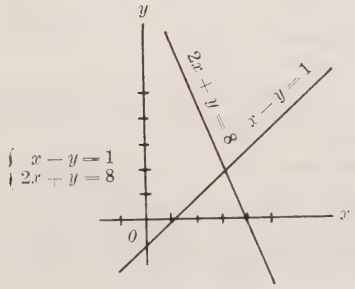
2. First Degree Relations

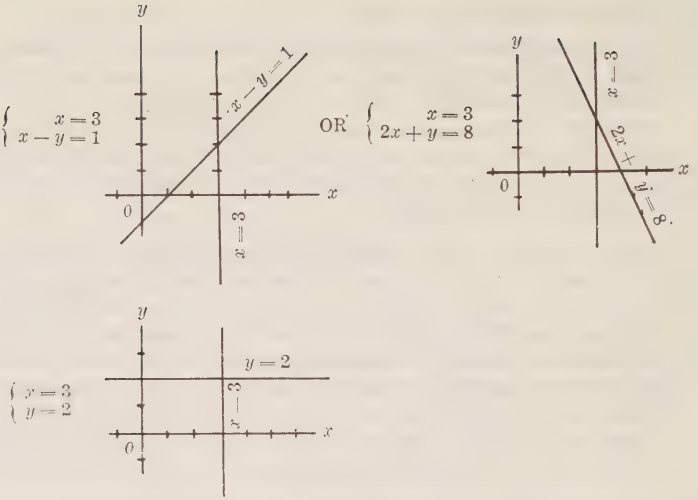
The student should appreciate that if x and y represent real numbers, the graph of $y = 3x + 6$ contains *every* point on a straight line and the graph of $y > 3x + 6$ contains *every* point in a region. Students should also graph simple relations which are not functions such as $x = y^2$. The vertical line test for a function should be introduced at this time.

Students will need considerable practice in graphing first degree relations and determining intercepts, domain and range. Graphs of relations involving union and intersection provide interesting practice and lead to the solutions of equations in two variables. It would be interesting to do some simple problems in linear programming at this stage to show an application of inequality graphs. The use of colour is suggested to shade in regions in inequality graphing.

3. In section 2, students will have learned to express equations in the form $y = mx + b$ and to solve a system of equations in two variables graphically. It is important to note that at the point of intersection the ordinates are equal and that this leads to the method of solving by comparison. The method of substitution follows when the point of intersection is written in the form $(x, 3x + 6)$ and it is appreciated that the coordinates of this point satisfy the second equation.

In developing the procedure for solving a system of two first degree equations in two variables by the method of elimination by addition or subtraction, the graphical interpretation of the equivalent systems in the solution should be emphasized, e.g.,





In problem solving, students should be taught to analyse each problem carefully rather than to memorize procedures for "type problems". The emphasis should be on the development of the necessary equations. Students should be encouraged to examine their solutions to the problem so that "unreasonable" answers caused by errors can be recognized.

The use of 2×2 matrices in solving two equations in two variables would be an interesting lesson to present another method of solving equations in two variables and would be useful to the students in their later work.

In Section 4, it is assumed that the work of Grade 9 will be reviewed at the proper time as work proceeds in the analytic geometry of the straight line. Meaning could be given to the idea of a slope by comparing it to the slope of a roof or a hill. The role of m and b should be *discovered* by plotting several lines (a) having the same m and (b) having the same b.

It is not intended that students learn several formulas for the equation of a line but that equations be obtained from the basic idea of constant slope, i.e., $\frac{y - y_1}{x - x_1} = m$

Due to the length of time allotted to this section, involved or tricky problems should be avoided. However, many useful ideas involving length, slopes and inter-

sections could be attempted: e.g., a particular case showing that the line segment joining the mid-points of the sides of a triangle is parallel to the base and equal to one-half the base. It is necessary that students have a good understanding of first degree equalities and inequalities in two variables and their graphs as well as basic ideas on relations and functions.

UNIT 3
GEOMETRY

1. Deduction in Geometry

This topic presents a logical extension of the work in earlier grades with a greater emphasis on deduction. It is the intent with geometric properties of plane figures established inductively in Grades 7, 8 and 9 be accepted as the basis for the unit. This previous work includes:

- intuitive acceptance of the ideas of point, line, plane, ray, line segment, angle and parallelism
- equality of vertically opposite angles
- angle sum of a triangle
- angle relationships in isosceles and equilateral triangles
- relationship between an exterior angle and the interior angles of a triangle
- the equality of alternate and of corresponding angles formed when a transversal crosses two parallel lines
- congruence of triangles: side-side-side, side-angle-side, angle-side-angle

These ideas and properties should be quickly reviewed without any attempt to prove them deductively.

Deduction should be introduced, and used to proceed to the development of further properties of plane geometry. Short sequences of related theorems or deductions should be developed, e.g., starting from the angle sum of a quadrilateral, a pentagon, and an n-gon can be deduced; starting from congruence and the angle equalities associated with a transversal and parallel lines, the properties of parallelograms can be deduced.

Emphasis should be placed on the oral analysis and solution of deductions. In an oral discussion of a problem, the student should not be expected to state all authorities but should be prepared to be asked by others in the class to justify any statement made.

Written proofs of deductions should be in good form and should include sufficient authorities to indicate that the reasoning is sound.

The symmetry of geometric figures should be discussed as a motion that brings the figure into coincidence with itself — either by means of a reflection in a line (the axis of symmetry) or by means of a rotation through a given angle.

A selection from the following list of figures is suggested for study:

isosceles triangle
parallelogram
rectangle
equilateral triangle
square
rhombus

The ideas of group structure can be reinforced by a consideration of the group symmetries of one of the regular polygons.

2. Three-dimensional Geometry

Three-dimensional geometry is the study of geometric figures and their relation to each other in space. Extensive use of models and neat three-dimensional diagrams are essential.

The time allowed permits little more than a brief introduction to this topic and should be treated as an extension of the study of geometry in the plane. Definitions should be constructed to parallel those of plane geometry.

Students should have some practice drawing three-dimensional diagrams. This work may be introduced when discussing relationships between points, lines and planes in three dimensions.

A discussion of the five regular polyhedra should include:

(i) use of models

(ii) definition of polyhedron, face, edge, vertex, and regular polyhedron; the names of the five regular polyhedra

(iii) discovery of Euler's relation

3. TRANSFORMATIONS

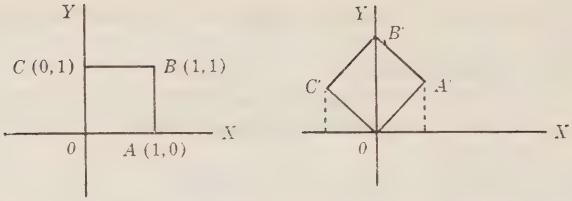
A transformation is a 1:1 matching of sets of points. The simplest cases are translations, reflections and rotations. They can be thought of as motions of points, line segments and geometric figures, and illustrated with two- and three-dimensional models.

The subject of geometry may be defined as the study of properties of geometric figures which are invariant (unchanged) under different transformations. For example, in this topic, it is established that the above-mentioned transformations are isometries, i.e., length, shape and size are preserved, and that the image is congruent to its pre-image under these motions. This approach to geometry was first conceived by Felix Klein in 1872 and developed in his famous Erlangen program. The underlying purpose is to distinguish the important properties of geometric figures from the trivial ones by noting those which remain unchanged when the figure is altered or moved in some way.

The introductory work of this section could be done on graph paper in a Cartesian coordinate system. Translations might be written as column vectors, in order to distinguish them from the positions of points, and their compositions used to establish group properties.

Reflections may be illustrated by paper folding, the use of carbon or tracing paper, as well as with sketches and diagrams. The relationship to physical reflection in a mirror should be brought out.

In the case of rotations of the square and equilateral triangle about a vertex, the calculation of lengths of diagonals and altitudes afford a review of the Pythagorean theorem and square root: e.g., "A square has vertices 0 (0, 0), A (1, 0), B (1, 1), C (0, 1). Find the images of its vertices under a rotation of the figure about 0 through a counter-clockwise angle of 45°."



Throughout this section the use of physical analogies to the mathematical concepts is important. The constructive, physical, intuitive approach with stress on the "motions" of the geometric figures is desirable.

A dilatation is the transformation of the plane which for a given point O assigns to every point P an image point P', such that $OP' = k \cdot OP$, $k \in R$. The pantograph is an instrument which can be used to illustrate the basic idea. As mentioned above, the case of graphs and coordinates would be helpful at the beginning when the properties of the sides of similar figures are established. It is not intended that there be any extensive development of theorems and deductions.

The term "group" when applied to a set of transformations, implies that the successive application of two transformations of the set results again in a transformation of the same set, and that the inverse of a transformation of the set again belongs to the set. The group properties of successive transformations of one type, applied to a figure, such as rotations of a square, are simple and obvious. The more interesting cases involve mixtures of types.

An example is the group of transformations of an equilateral triangle under rotations through 120°, 240°, and reflection about (or rotation through 180°) a specific axis of symmetry.

4. VECTORS

The purpose of the section is to give the pupil intuitive grasp of the notion of vectors, the distinction between the class of equivalent segments (or ordered pairs) and the individual representation used in a diagram, and the usefulness of the concept. The purpose is not to develop an algebra of vectors rigorously, nor to apply the vector approach to difficult deductions.

The work on translations in the preceding section may be used to introduce vectors. The definition of vector as an ordered pair (or triple) of numbers, and as an infinite set of equivalent directed line segments can be introduced simultaneously. The geometric vector then becomes the individual directed line segment used as a representation for the infinite set, or ordered pair.

At the outset it is important to have the pupil see the relationship of vector addition to the addition of real numbers. The use of the number line and review of the representations which illustrated the addition of positive and negative numbers should be helpful.

An important application of vector addition is the determination of the effect of wind on the speed and "track" of an aircraft, and the effect of currents on the course of a ship. In these cases a polar method of representation of vectors, involving the direction and length of the line segment, is generally used. This will have to be explained; for example, in the case of the vector (1, 1), the length of the line segment is $\sqrt{2}$ and its angle with the vertical is 45°. If we use compass directions (clockwise in degrees from north), then the vector might be designated by (45°, $\sqrt{2}$).

A good example to illustrate the application of vector methods to geometric deductions is the theorem: "The line segment joining the mid-points of the two sides of a triangle is parallel to the third side and equal to one-half of it." Examples should be chosen with care; it should be pointed out that this approach is not the only one.